AJEB
journal.amca2012.org

AMCA JOURNAL OF EDUCATION AND BEHAVIORAL CHANGE
e-ISSN 2776-1185
Vol 1, No 2, October 2021, 61-68
DOI 10.51773/ajeb.v1i2.124

Students' mathematical reasoning in mathematics problems solving

Baiduri[1*]

[1]University of Muhammadiyah Malang, Indonesia
[*]Corresponding author: baiduri@umm.ac.id

KEYWORDS
Mathematical reasoning
mathematical problems
problem solving

ABSTRACT This study aims to describe how the students' interest in learning class VIII SMP after using a traditional game guide book with the selection of subjects who have low interest in learning mathematics. The research used a descriptive form with a qualitative approach. The data from this research are students' interest in learning. The object of this study were students of class VIII SMP. The data collection techniques used were questionnaires and interviews as data reinforcement from the questionnaire results. The results of this study indicate that the use of traditional game guidebooks in mathematics learning is very effective, as seen from the results of the questionnaire on student interest in learning where male students' interest in learning increases by 79 This study aims to explore and describe students' mathematical reasoning and the level of reasoning in solving mathematical problems TIMSS type with level intermediateand and the cognitive domain of reasoning. Mathematical reasoning in this study is categorized into analysis (A), generalization (G) and justification (J). The data were obtained from the written answers of 27 seventh grade students of a religiously minded junior high school and interviews with a representative of the type of answer group that was the subject of the research. Data analysis was carried out based on students' written answers and interviews by categorizing mathematical reasoning and levels. The results of data analysis prove that mathematical reasoning in the analysis category occurs when understanding the problem and making a resolution plan is at the consolidation level, and at the expanding level when solving problems. Generalization is done when solving the problem is at the developing level. While justification is done when understanding the problem, when solving problems and re-checking the results of the answers. Student justification is at the developing level. These results show that mathematical reasoning is inseparable in solving mathematical problems.

1. INTRODUCTION

Mathematics is one of the oldest disciplines and offers valid content for the school curriculum. Knowledge of mathematics is taught with the hope that learning mathematics, in addition to improving reasoning and cultivating the mind in general, can provide students with a systematic way to approach various problems and as a tool for analyzing and modeling situations and events in the physical, biological and social sciences. Mathematical reasoning and mathematical proof are fundamental aspects of mathematics (Mata-Pereira & da Ponte, 2018). The reasoning process that is central to mathematics education is shaped by preschool experience and is also influenced by activities outside of school. The reasoning abilities developed in mathematics are applied to the learning of other subjects, while experiential learning in other fields can advance the development of mathematical reasoning (Nunes & Csapó, 2015). The importance of assessing students' reasoning is to understand how students form generalizations and why they think their mathematical statements are true (Lannin et al., 2011).

Reasoning is defined as a line of thought, a way of thinking, which is adopted to produce affirmations and reach conclusions (Bergqvist & Lithner, 2012; Lithner, 2008). Reasoning is explicitly stated as a skill to be developed in students and is defined as '... the capacity for logical thinking and action, such as analyzing, proving, evaluating, explaining, inferring, justifying and generalizing' (Australian Curriculum Assessment and Reporting Authority, 2015). Reasoning is a fundamental component in learning mathematics (Bergqvist & Lithner, 2012; Fatimah & Prabawanto, 2020; Lithner, 2008; Masfinatin et al., 2020; Thompson et al., 2012), because it includes a variety of mathematical activities that involve higher order thinking (Melhuish, 2020). If reasoning skills are not developed in students, mathematics becomes a matter of following a series of procedures and imitating examples without thinking about why they make sense (Melhuish, 2020; Nunes & Csapó, 2015).

Mathematical reasoning cannot be separated from knowing mathematics with understanding (Tajudin & Chinnappan, 2015). Mathematical reasoning is part of the problem solving process that involves students' thinking and reasoning skills in finding alternative problem solving (Niswah

& Qohar, 2020; Sandy et al., 2019). Students' mathematical reasoning ability can solve mathematical problems in the learning process. Problem solving tends to be the focus when examining or supporting students' mathematical reasoning (Segerby & Chronaki, 2018). Problem solving is not only a valuable competency in students, but also a way to approach mathematics and achieve other goals namely mathematical reasoning, which is important and improves mathematical understanding (Jäder et al., 2020; Lithner, 2008).

Several previous studies related to students' mathematical reasoning in problem solving (Rachmaningtyas et al., 2020; Rahmawati et al., 2018; Sitrava, 2019; Vale et al., 2017) and mathematical reasoning in the learning process (Jeannotte & Kieran, 2017; Masfinatin & Murtafiah, 2020; Niswah & Qohar, 2020; Salam & Salim, 2020; Zakaria & Amidi, 2020), classification of students' reasoning competence in problem solving (Rachmaningtyas et al., 2020) and the relationship between mathematical reasoning and problem solving in the learning process (Kusuma et al., 2020). The tendency of these researchers to use indicators or categories of mathematical reasoning is different and has not been analyzed at the stage of problem solving. Therefore, the research aims to describe and explore students' mathematical reasoning with categories analysis, generalization, and justification in the stages of solving mathematical problems.

2. MATHEMATICAL REASONING

Reasoning in mathematics is the process of applying logical thinking to a situation to derive the correct problem-solving strategy for a given question, and using this method to develop and describe a solution. Simply put, mathematical reasoning is the bridge between fluency and problem solving. Reasoning competence means being able to carry out mathematical reasoning involving concepts and methods to form solutions to problems and modeling situations (Segerby & Chronaki, 2018). The essence of the reasoning process is the student's ability to make conclusions that are justified with conjectures, generalizations, and justifications (Mata-Pereira & da Ponte, 2018; Mukuka, 2020). The process of mathematical reasoning includes formulating questions and problem-solving strategies, formulating and testing generalizations and other conjectures, and justifying them (Mata-Pereira & da Ponte, 2017)

The categorization of mathematical reasoning is very diverse. Nevertheless, there are many similarities. Jeannotte & Kieran (2017) identify two aspects of mathematical reasoning, namely: structural aspects and process aspects. The structural aspect refers to the way in which the discursive elements are joined in an orderly system that describes the various elements and the relationships between the elements. Forms of mathematical reasoning based on structural aspects are deductive and induction reasoning. Aspects of the mathematical reasoning process is a meta-discursive process, that is, derived narratives about objects or relations by exploring the relations between objects. Based on the aspect of the process, mathematical reasoning is classified into two categories: processes related to finding similarities and differences, or processes related to validating (Jeannotte & Kieran, 2017). While the process for validating, mathematical reasoning is categorized into validating, justifying, and proving (Jeannotte & Kieran, 2017). Some other researchers categorize mathematical reasoning into: justification, generalizing, and using procedures/facts (Melhuish, 2020), generalizing, justifying, comparing, classifying and exemplifying (Rodrigues et al., 2021), analyzing, forming conjectures and generalizing, and justifying and logical arguments (Australian Government Department of Education and Training, 2017), comparing and contrasting, generalizing, and justifying (Vale et al., 2017), validity, generalizability and efficiency of solutions (Sitrava, 2019). Validity regarding the correctness of student answers. Generalization has to do with whether the strategy works for a different problem with the given problem. While efficiency is related to how and when strategies can be used more efficiently. Mathematical reasoning in this study refers to the analysis, generalization, and justification in accordance with (Davidson et al., 2019; Loong, 2018) because it is very simple and suitable for basic education.

Analyzing involves exploring a problem using a given example or generating an example to form or test a conjecture. Analyzing occurs by comparing and contrasting cases, what is the same and what is different, what has changed in order to sort and classify cases. Analyzing involves using numerical or spatial structures, known facts or properties when sorting cases or repeating and expanding patterns (Australian Government Department of Education and Training, 2017; Loong, 2018). Case categories and patterns are identified by labeling using terms, diagrams or symbols.

Generalization is a process that concludes a narrative about a set of mathematical objects or the relationship between a set of objects from a subset (Jeannotte & Kieran, 2017). There are four essential understandings of generalizations: (1) developing statements, (2) identifying similarities and extending the original case, (3) recognizing domains that hold generalizations and (4) clarifying the meaning of terms, symbols and representations (Lannin et al., 2011). Generalization involves identifying a common trait or pattern in more than one case and communicating the rules (guess) to describe the trait, pattern or relationship (Australian Government Department of Education and Training, 2017; Loong, 2018; Vale et al., 2017). As the basis of mathematical concepts and ideas, generalization is central to the reasoning process (Mata-Pereira & da Ponte, 2017) so it is important to involve students in situations that encourage generalization because a lot of mathematics learning can develop from these activities. However, we should note that generalizations in class have not gone as expected (Hjelte et al., 2020; Mata-Pereira & da Ponte, 2017). Generalization is the capacity to communicate thoughts (Vale et al., 2017).

Justification is structural for proof and, therefore, very important for the development of students' mathematical knowledge. Justification allows students to imagine mathematics as a logical, interrelated and coherent subject (Mata-Pereira & da Ponte, 2018). Justification allows students to understand mathematics for themselves and convince others that the procedures or strategies they are using are valid or just conjectures or generalizations are justified (Carpenter et al., 2003; Lannin et al., 2011). Justification involves checking the veracity of conjectures and generalizing to show or disprove the truth of a claim using logical arguments (Australian Government Department of Education and Training, 2017; Loong, 2018; Vale et al., 2017). To do so, justification depends on accepted mathematical concepts, properties, procedures, and ideas. Therefore, it is important for students to understand the need for justification from an early age in their schools. Students should engage in justifying by relying on previously understood mathematical ideas or disproving statements by providing counter-examples. It is also important for students to develop knowledge of what validates an argument or refutes an argument.

3. PROBLEMS SOLVING

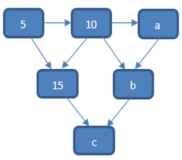
The importance of problem solving in learning mathematics stems from the belief that mathematics is not about memorizing, but about reasoning. Problem solving allows students to develop understanding and explain the processes used in finding solutions, rather than remembering and applying a series of procedures. Problem solving competence, mathematical reasoning, procedural fluency, and conceptual understanding are indicators in mastering mathematics (Jäder et al., 2020). The problem solving process may also include the use of competencies such as procedural fluency, which will encourage a deeper understanding of mathematics. Mathematical problem solving involves a complex set of processes – identifying the problem (understanding), interpreting what to do (planning), selecting and implementing a problem-solving strategy (execution) and then assessing the reasonableness of the solution (looking back) (Baiduri et al., 2020; Saundry & Nicol, 2006). There is a positive relationship between mathematical reasoning and problem solving ability (Tajudin & Chinnappan, 2015).

4. METHOD

The purpose of this research is to explore and describe the students' mathematical reasoning process in solving TIMSS type math problems. The type of research used is descriptive exploratory with a qualitative approach (Creswell, 2017). The participants of this study were students of class VIII SMP with an Islamic perspective, amounting to 27 people. The data needed in this research is students' mathematical reasoning in solving mathematical problems. Data were collected through students' written answers in solving given mathematical problems as well as from semi-structured interviews. Interviews were conducted with a representative of the answer group.

The instrument in the form of three essay test questions was developed referring to the Trends in International Mathematics and Science Study (TIMSS) type with algebraic content of two questions (numbers 1 and 2) and data content of one question (number 3). All questions with intermediate level and cognitive domains of reasoning were:

1. What are the values for a, b, and c in the picture below? Explain how to get it.



2. The image to the right shows the number of tiles 3x3, 4x4, and 5x5. Find the number of tiles that are blue and white in sizes 6x6 and 7x7 !

3. Look at the number pattern on the side. Determine the formula for the nth number!
$\frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \frac{7}{8}, \frac{8}{9}, \frac{9}{10}, \ldots$

While the questions asked for the analysis are; 1) What are the facts in the problem?, 2) What are the characteristics of the problem?, 3) What will be done to solve the problem?, 4) What do you pay attention to?, and 5) What do you think will happen next? if you do this?. For generalization, the questions asked are; 1) What is the pattern/trait here?, 2) Does the pattern always work?, 3) What happens in general?, 4) What are the rules?, 5) Are there other examples that fit the rules?, and 6) How can you describe the pattern? As for justification, the question is; 1) Convince me, that what you are doing is right? 2) How can I be sure...? How do you know...?, and 3) Explain - why (process/procedure/outcome) is true? (Australian Government Department of Education and Training, 2017).

The data analysis was conducted on the interview results and written answers of students to the three questions by categorizing the students' mathematical reasoning and then being coded according to the possible level of reasoning. The three categories of mathematical reasoning are analysis (A), generalization (G) and justification (J) as well as possible levels of reasoning of each category; unclear (A0, G0,J0), beginning (A1, G1,J1), developing (A2, G2,J2), consolidate (A3, G3,J3), and expand (A4, G4,J4) (**?**)(Davidson et al., 2019; Loong et al., 2018). As the basis for the analysis used indicators of the mathematical reasoning process in this study are presented as the following list (Davidson et al., 2019; Loong, 2018), while the indicators for each level of reasoning are presented in Table 1. The indicators of mathematical reasoning in problem solving are:

1. Mathematical Reasoning
   - Indicators of Mathematical Reasoning Process in problem solving
2. Analysis
   - Identify the facts in the question
   - Identify traits
   - Identify the solution strategy/formula
3. Generalization
   - Forming conjectures about general properties
   - Expanding common traits
   - Generalizing properties
   - Forming associations between two or more existing problems or situations or objects
   - Looking for the same relationship, procedure, pattern, solution or result
4. Justification
   - Verifying the truth of the facts on the question
   - Verify the correctness of the similarity of properties or procedures
   - Verifying the correctness of the completion strategy
   - Explain the truth of the answer

TABLE 1. Levels and indicators of mathematical reasoning in problem solving.

Level	Analysis	Generalization	Justification
Not clear	Not paying attention to the nature or pattern	Does not communicate general traits or rules	Do not justify
Beginning	Remembering randomly known facts or attempts to sequence repeated examples or patterns	Attempting to communicate a general trait or rule to a pattern	Explain what they do and recognize what is right or wrong
			Arguments are incoherent or do not cover all steps
Develop	Paying attention to generality, or sorting and listing cases, or repeating and expanding patterns	Communicating rules using mathematical terms and noting other cases or examples	Attempts to verify by testing cases and detecting and correcting errors or inconsistencies
	Describe the nature or pattern		The initial statement in the logical argument is true
Consolidation	Systematically look for examples, expand patterns or analyze structures to form conjectures	Communicating the rules using mathematical symbols and explaining what the rules mean or explaining how the rules work using examples	Verifying the truth of the statement by confirming all cases or refuting the claim using counter examples
	Making predictions about other cases		Using correct logical arguments
Expand	Observe and exploit relationships between traits	Patterns or properties using mathematical symbols and applying rules.	Using unquestionable logical arguments
		Compare different expressions for the same pattern or trait to show equality	Verify that generalizations apply to all cases using logical arguments

- Extending generalizations using logical arguments

5. RESULTS

To analyze students' mathematical reasoning in the problem solving process based on written answers and strengthened by interviews based on written answers.

5.1 Solving problem 1

When understanding the problem, students analyze the pattern of numbers given, right and down. For the pattern of numbers to the right, there are two patterns found by students, namely number multiples (multiplication by 2) which is presented in solution (1) and multiplication of number 5 (the next number is obtained by adding 5) such as completion (2). While the pattern of numbers below, obtained by adding up the two numbers above. This was understood by all students who answered. Based on the analysis when understanding the problem, students expand the similarity of patterns or make conjectures about the nature to answer questions. Students' written answers are presented in Figure 1. Solution types (1) was carried out by three students, (2) was completed by 23 students, and one student did not answer.

To explore students' mathematical reasoning in depth, an interview was conducted with one of the students in each group of completion as follows.

R : What do you understand from the question?
P(1) : The arrangement of numbers, to the right of the number in front of it multiplied by 2, 10 = 5 x 2. The number below is obtained from the sum of the two numbers above, 15 = 5 + 10
P(2) : The order of numbers increases by 5 to the right, 10 = 5 + 5 and down adds up the two numbers above it, 15 = 5 + 10
R : Are you sure the order of numbers is as stated?
AP : Yes, because it's been researched
R : What did you do to answer the question in the question?
P(1) : Using this form, a = 10 x 2 = 20; b = 10 + 20 = 30, and c = 15 + 30 = 45
P(2) : Following this pattern, a = 10 + 5 = 15; b = 10 + 15 = 25, and c = 15 + 25 = 40
R : Are you sure your answer is correct?
AP : Sure
R : Explain, why the results you get are correct?
AP : Because it follows the previous form, and the calculation is correct

5.2 Problem solving 2

When understanding the problem, students analyze the number of white and blue tiles for each given tile size. Although the number of tiles obtained is the same, the method of obtaining them is different. There are two different ways that students do to get white tiles, namely by adding (answer groups 2 and 3) and multiplying (answer groups 1 and 4). Likewise, to get a lot of blue tiles, there are two strategies that students use, namely multiplication (square) and addition (answer groups 1 and 2) and using the difference between many tiles and many white tiles (answer groups 3 and 4). Based on the analysis when understanding the problem, students expand the similarity of patterns or make guesses about the nature of making plans for solving and answering questions, except for one group of students (completion type 2) which is inconsistent with the results of the analysis when understanding the problem. When understanding the problem, students of completion type 2 get white and blue tiles by adding up, while in planning their solution they are based on patterns. Students' written answers are presented in Figure 2. Solution types (1) was carried out by twelve students, (2) was carried out by four students, (3) was carried out by one, (4) was carried out by nine students and one student did not answer. Students feel sure the answer is correct because it is in accor-

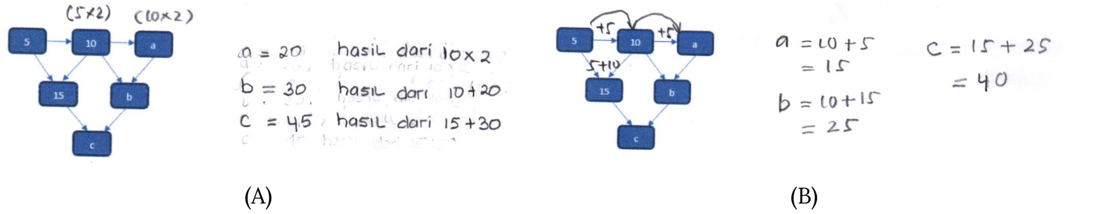


FIGURE 1. Types of student's solution in question 1. Figure (A) solution types was carried out by three students, and figure (B) solution types was completed by 23 students

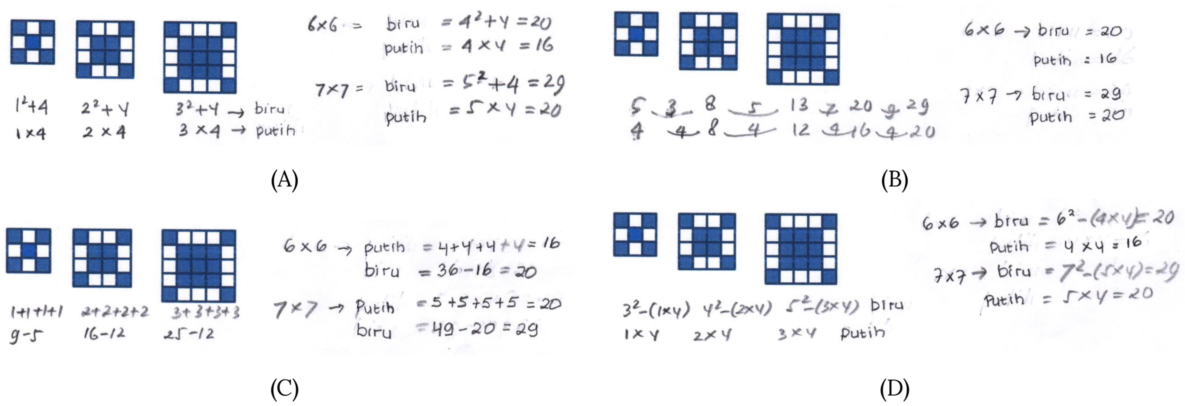


FIGURE 2. Types of student's solution in question 2. Figure (A) was carried out by twelve students, (B) was carried out by four students, (C) was carried out by one, (D) was carried out by nine students and one student did not answer.

dance with the previous plan and the arithmetic operation has been checked.

Based on the type of written answer in Figure 2, to explore students' mathematical reasoning in depth, an interview was conducted with one of the students in each completion group as follows.

R : What do you understand from the question?
AP : There are tiles, squares in white and blue
R : How many tiles are there? Why do you say square?
AP : Three, because the sizes are 3 x 3, 4 x 4, and 5 x 5
R : How many colors are white and blue in each square?
AP : Different for each tile. White; 4, 8, 12. Blue; 5, 8, 13
R : What else do you understand from the question?
AP : Will look for lots of blue and white tiles at 6 x 6 and 7 x 7
R : Are you sure what you understand is correct? Why
AP : Yes, because it's already in the problem.
R : Earlier you mentioned the number of white tiles for size 3 x 3 is 4, size 4 x 4 is 8, and size 5 x 5 is 12, How do you get them?
P(1) : Size 3 x 3, 4 = 1 x 4, size 4 x 4, 8 = 2 x 4, and size 5 x 5, 12 = 3 x 4
P(2) : Add up the white tiles, 4 = 1+1+1+1, 8 = 2+2+2+2, and 12 = 3+3+3+3
P(3) : Add up the white tiles, 4 = 1+1+1+1, 8 = 2+2+2+2, and 12 = 3+3+3+3
P(4) : Size 3 x 3, 4 = 1 x 4, size 4 x 4, 8 = 2 x 4, and size 5 x 5, 12 = 3 x 4
R : What do the numbers 1, 2, 3 and 4 in multiplication represent?
P(1), P(4) : Numbers 1, 2, and 3 have a lot of white tiles on one side, step 4 because there are many sides on the original tile that have 4 sides.
R : If a lot of blue tiles, how do you get them?
P(1) : Size 3 x 3, 5 = 12 + 4, size 4 x 4, 8 = 22 + 4, and size 5 x 5, 13 = 32 + 4
P(2) : Add up the blue tiles, 5 = 1+1+1+1+1, 8 = 4+1+1+1+1, and 13 = 9+1+1+1+1
P(3) : Number of tiles minus white tiles, size 3 x 3, 5 = 9 − 4. Size 4 x 4, 8 = 16 − 8. Size 5 x 5, 13 = 25 − 12.
P(4) : Size 3 x 3, 4 = 32 − (1 x 4), size 4 x 4, 8 = 42 − (2 x 4), and size 5 x 5, 13 = 52 − (3 x 4)
R : When you determine the number of blue tiles, there are 12 + 4, 22 + 4, and 32 + 4. What does the square number represent? and the number 4 says what?
P(1) : 12, 22 , 32 represent the number of blue tiles in the middle, and step 4 the number of blue tiles in the corners.
R : When you determine the number of blue tiles, 4 = 32 − (1 x 4), 8 = 42 − (2 x 4), and 13 = 52 − (3 x 4). What does the square number say? and what does the multiplication number in brackets say?
P(4) : 32, 42 , 52 represent the total number of tiles according to their size. The multiplication number in brackets is the number of white tiles
R : What would you do to answer what was asked?
P(1,3,4) : Following how to search on the previous tile
P(2) : The white and blue tiles already have a pattern. White tiles 4, 8, 12 and so on. Blue tiles 5, 8, 13 and so on
R : How do you find the number of white tiles at

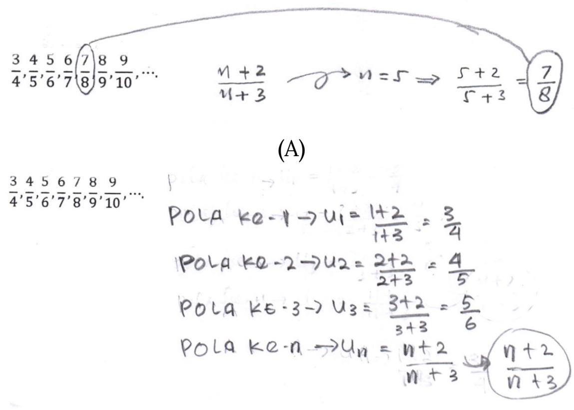
6 x 6 and 7 x 7?
P(1) : Like looking at the previous size. Size 6 x 6, 4 x 4 = 16, size 7 x 7.5 x 4 = 20
P(2) : Following the pattern, the white tiles on 3 x 3, 4 x 4, and 5 x 5 are 4, 8, 12. The difference, 4. For size 6 x 6 plus 4, so 16 and size 7 x 7 plus 4 again becomes 20
P(3) : Like looking for the previous white tile, the size is 6 x 6, the white tile is 4+4+4+4 = 16, the size is 7 x 7, 5 + 5 + 5 + 5 = 20.
P(4) : As before, Size 6 x 6, 4 x 4 = 16, and Size 7 x 7.5 x 4 = 20
R : If there are a lot of blue tiles, how do you find them at 6 x 6 and 7 x 7?
P(1) : Like the previous pattern, for size 6 x 6, 42 + 4 = 20, size 7 x 7, 52 + 4 = 29
P(2) : Following the pattern, the blue tiles at 3 x 3, 4 x 4, and 5 x 5 are 5, 8, 13. 5 to 8 plus 3, 8 to 13 plus 5, meaning for size 6 x 6 plus 7, so 20 and size 7 x 7 plus 9 more to 29
P(3) : As before, number of blue tiles = number of tiles minus white tiles, size 6 x 6, 36 − 16 = 20. Size 7 x 7, 49 − 20 = 29.
P(4) : Same as before, size 6 x 6, 62 − (4 x 4) = 20, size 7 x 7, 72 − (5 x 4) = 29.
R : Are you sure your answer is correct? **AP** : Sure
R : Explain, why the results you get are correct?
AP : Because it follows the previous form, and the calculation is correct

5.3 Problem solving 3

When understanding the problem, students analyze the pattern of numbers in the numerator and denominator, each of which is increased by one. Based on the analysis when understanding the problem, students expand the similarity of patterns or make conjectures about the nature to answer questions. The students' written answers are presented in Figure 3. Solution types (1) was carried out by eleven students, (2) was carried out by twelve students, and four students did not answer (Table 3). Students feel sure the answer is correct because it is in accordance with the pattern obtained previously and the arithmetic operation has been checked. Based on written answers and interview results, when understanding the problem, all participants who answered did an analysis of the number pattern in both the numerator and denominator. Generalization by using patterns or relationships that are already known when solving problems. While the justification is done by re-checking the existing pattern on the problem when understanding the problem, using the previous formula when solving the problem and re-checking the answers, and checking the results of arithmetic operations when rechecking the answers.

Based on the type of written answer in Figure 3, to explore students' mathematical reasoning in depth, an interview was conducted with one student in each group of completion as follows.

R : What do you understand from the question?
AP : Fractional arrangement.
R : How are the numbers arranged?
AP : The top is increased by 1, the bottom is also



(B)

FIGURE 3. Types of student's solution in question 3. Figure (A) was carried out by eleven students, (B) was carried out by twelve students.

added by 1
R : What do you mean increase by 1?
AP : The top one, 3, 4, 5, 6, 7, 8, etc. The bottom 4, 5, 6, 7, 8, 9, etc.
R : What else is understood from the question?
AP : Asked to find the formula for the number n
R : How to find it?
P(1) : Because there is already a pattern, then just go straight to the formula, $\frac{n+2}{n+3}$
P(2) : Look for the pattern first, 2nd pattern, 3rd pattern, and nth pattern. The patterns are: $U_1 = \frac{1+2}{1+3} = \frac{3}{4}; U_2 = \frac{2+2}{2+3} = \frac{4}{5}; U_3 = \frac{3+2}{3+3} = \frac{5}{6}; U_n = \frac{n+2}{n+3}$
R : Why add 2 to the numerator and 3 to the denominator?
AP : Because for the numerator, 1 + 2 = 3, 2 + 2 = 4, 3 + 2 = 5 and so on. The numerator is increased by 2. For the denominator 4 = 1 + 3, 5 = 2 + 3, 6 = 3 + 3 etc. The denominator increases by 3.
R : Are you sure the answer is correct?
P(1) : Sure true, for example n = 5, then $\frac{5+2}{5+3} = \frac{7}{8}$; which is the same as the 5th form as in the problem
P(2) : Sure, because the requested nth pattern is obtained from the previous pattern

Note: R: Researcher; P(i): The representative of the participant from the answer group type i; AP: All participant representatives from the answer type group

6. DISCUSSION

From the written answers and interview results, the analysis is carried out when understanding the problem and making a settlement plan by observing the data provided, analyzing patterns to build a guess. Meanwhile, when solving problems, to obtain the desired answer, students use and explore the relationship between the previous pattern. Thus the level of mathematical reasoning at the time of analysis is at the level of consolidation when understanding the problem and making a resolution plan, and the level of expanding when solving problems (Davidson et al., 2019; Masfinatin et al., 2020).

Generalization is done when solving problems using known patterns or relationships, communicating rules using mathematical terms, recording other cases or examples, expanding common trait, generalizing properties, and looking for the same relationship, procedure, and pattern. This shows that students understand how to justify generalizations by using valid arguments why the statement is true (Brunheira & Da Ponte, 2019). This means that students' mathematical reasoning when generalizing is at the developing level (Davidson et al., 2019; Masfinatin et al., 2020). In addition, students also identify similarities and expand the original case in finding solutions (Lannin et al., 2011; Melhuish, 2020).

While the justification is done by re-checking the data in the problem when understanding the problem, using the formula that existed previously when solving the problem and re-checking the results of the answers, and checking the results of arithmetic operations when re-checking the answers. Justifications made by students through explanations of what has been done are related to the correctness of their work and verifying and detecting and correcting if there are any that are inconsistent. Justifications made by students are based on existing patterns or empirical evidence (Lannin, 2005). Justification for students in solving problems is at the level of developing (Davidson et al., 2019; Loong, 2018).

7. CONCLUSION

Mathematical reasoning and problem solving are two very important things in learning mathematics and mathematics. Mathematical reasoning can be trained with problem solving, problem solving requires a logical thinking process or reasoning. The category of mathematical reasoning in this study includes analysis, generalization and justification in solving TIMSS type mathematical problems. The analysis category is carried out when understanding the problem, making plans and solving problems. The generalization category occurs when solving problems by identifying similarities and expanding cases to find solutions. Justification occurs when understanding the problem, solving the problem and re-checking the answers.

Further research can be done using different reasoning categories or TIMSS questions with elementary or advanced levels.

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