

## Students' misconception and errors in solving relations and functions problems

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### KEYWORDS

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principal errors  
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**ABSTRACT** The objectives of this present research are as follows: 1) to analyze students' errors in solving the relations and functions problems, and 2) to describe students' misconceptions of the relations and functions materials. A descriptive qualitative approach was adopted. The data source was 23 students of the grade VIII-A MTs Muhammadiyah Malang Indonesia. The data investigated were the students' errors and misconceptions of the Relations and Functions materials. In this research, the subjects were determined using a purposive technique where 6 students were chosen. They were divided into three categories, namely students with high, low, and low abilities, where each category consists of two students. Two techniques of data collection were employed: 1) administration of written tests to students; and 2) interviews. The technique used in the data analysis was Miles and Huberman's model analysis. The research results showed that: 1) the subjects with high ability less properly stated a relation or a function in the form of an arrow diagram and demonstrated some misconceptions in giving examples of functions and non-function; 2) the subjects with low ability less accurately proved a relation and a function in the form of an arrow diagram, did not state the form of a sequential pair and showed technical and principal errors. And 3) subjects with low ability had all types of errors.

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### 1. INTRODUCTION

The development of science and technology requires someone to be able to master any information and knowledge as good as possible (Hasratuddin, 2014). In this science and technology era, it is greatly effortless to obtain any information and knowledge through various media such as printed media, electronic media, and books (Amir, 2015b). Consequently, it needs a great ability in selecting and processing information, and also a critical, logical, creative and consistent thinking action so that the information will be easily comprehended (Hasratuddin, 2014; Paladang et al., 2018). To develop such an ability, a program through education is required (Hasratuddin, 2014). Education is a system that establishes some components of educators, students, the goal of education, the tools of education, and the educational environment. (Arifin, 2015). It is through education that each student's ability and potency are developed to face any problems so that later she will become a high-quality, independent, and creative human being. Suhartini & Syahputra (2016); Anggreini & Waluyo (2017).

Mathematics is one of the ways to get an answer to a problem faced, namely the way: 1) to use information; 2) to use knowledge of shapes and size; 3) to use knowledge to calculate; and 4) the way to think of ourselves in perceiving and using relations (Mudilarto, 2003). Mathematics, with its feature full of abstract representations, is a collection of concepts, skills, and facts (Roselizawati et al., 2014). It is one of the subjects that greatly emphasizes concepts, and the

concepts themselves are something basic to be understood in a material (Irawati et al., 2014).

Some people think that mathematics is a branch of science that is difficult to understand (Malikha & Amir (2018); Paladang et al. (2018); Novitasari (2015) since it is abstract in nature (Novitasari, 2015). However, some others assume that mathematics greatly helps solve any problems in daily life (Novitasari, 2015). Hence, teachers are necessitated to discover and create learning strategies that enable students to interact well with their classmates and teachers and that arouse students' learning spirits.

Based on some researches, the results of learning relations and functions materials are still far from expectations. This condition results in low achievement in mathematics. Many students have problems in learning the functions materials; for instance, they are still unable to differentiate between functions and non-function expressed in the form of an arrow diagram, and some students are still difficult to solve essay problems dealing with functions (Kamariah, & Marlissa, 2016). But, it should be realized that one of the goals in learning the mathematics subject is to improve one's ability in solving any problems (Wahid et al., 2015) Efforts made to reach the goal is to introduce contextual problems in mathematics (Siswandi et al., 2016).

During learning activities, some students make errors in mathematics (Paladang et al., 2018). They do not always take up information fully so that sometimes a concept students grasp is different from what it should be (Syahrul & Setyarsih, 2015). Any conceptions which are not proper

with what should be are called misconceptions (Wahid et al., 2015). When a student makes an error and holds a misconception when he is solving a problem, the teacher should correct it so that such an error will not last longer (Raharjanti et al., 2016). Consequently, problems that often happen to students are not only errors but also misconceptions. Such misconceptions are caused by various factors such as the students themselves, the teachers, the learning contexts, the teaching techniques, and the textbooks used (Irawati et al., 2014). A misconception is a perception possessing meaning which is greatly different from the experts' opinion of a certain field (Hammer, 1996). It can also be said to be misunderstanding and misinterpretation from a meaning which is inaccurate (Ojose, 2015).

A student possesses a natural ability to think of various relations to recognize a certain object. It also happens in learning a science, where a student composes of the concept of knowledge by relating any knowledge he obtains in his daily life to new knowledge at school. A student's incomplete knowledge in the process of composing the concept may be caused by his limited ability. Therefore, it is this misconception that may happen to students. Misconception emerges from wrong experiences and beliefs from each individual. Then, each individual possesses different thoughts. In some researches, it is shown that one may not be able to distinguish between the terms error and misconception (Luneta & Makonye, 2010). The misconception is part of errors, meaning that one may define all misconceptions as errors, but not all errors might be misconceptions (Eryilmaz, 2002; Herutomo & Saputro, 2014). In other words, an error is the result of misconception, or misconception is a type of systematic perception that results in an error. For that reason, it is important for teachers to focus their attention on any misconception which is the source of errors (Eryilmaz, 2002). Incorrect concepts instigate students to make errors for concepts at further levels (Hidayati et al., 2016). Accordingly, teachers should identify any errors and misconceptions students possess, and they can correct them as soon as possible that the errors and misconceptions will not stay longer and will not influence students' next knowledge. Moreover, teachers should also focus more on improving or discovering proper models, methods, strategies, and approaches during learning processes (Herutomo & Saputro, 2014).

Any misconceptions some or all students experience should be gotten rid of since they may become wrong information and disturb students' further learning. Consequently, teachers call for some tools to detect any misconceptions students have through various techniques, namely: interviews, pre-concepts, essay texts, multiple-choice tests with reasons, discussions in the classrooms, or practices provided with question-answers. The purpose of identifying errors and misconceptions is to recognize the position of errors and misconceptions when solving problems related to Relations and Functions materials and to give more instructions or correct the errors and misconceptions existing in the students' minds so that they will not long last and become wrong knowledge in their minds. On the basis of the descriptions above, the objectives of this present research are as follows: (1) to analyze students' errors in solving Relations and Functions problems, and (2) to describe students' misconceptions of Relations and Functions materials.

## 2. LITERATURE REVIEW

### 2.1 ABILITY TO SOLVING PROBLEMS

The ability to solve problems is one of the basic competencies in mathematics a student should possess. Through a problem-solving type test, it is expected that a student will be able to develop his ability in solving mathematical problems. A student may be said to be successful in learning anything useful except he has some ability in using any information of knowledge he gains and ability to solve problems (Slavin, 2006). Problem is a problem if a student does not have any certain idea that may be used to get an answer to a question. Mathematical problems always deal with a question or a problem. Problem is often stated in the story, essay, or description forms, but not all story problems are problems. Story problems are not the same as problems because they are just merely media to express a problem. A question may become a problem if and only if it shows a challenge that cannot be solved using a routine procedure familiar with by the students (Shadiq, 2004). To solve a problem, one should choose the data of the information obtained and arrange any concepts possessed.

Problem in mathematics is a question or a problem that should be answered by each student and this needs deeper thinking and reasoning to find the answer (Farida, 2015). Posing problems routinely is one of the ways to improve student's ability in solving mathematical problems. It is stated that a problem usually contains a situation encouraging someone to solve it. Problem-solving is a process containing a lot of steps to find relationships between past experiences and a problem being faced and to do some actions to solve it (Rob, 2003). According to Polya, problem-solving is an effort to discover or look for a way out of any difficulty in attaining a goal that cannot immediately be achieved (Pólya, 1946). Problem-solving is an attempt used to solve problems; in this case Polya suggests four steps that should be taken: 1) understanding the problems, where students are asked to repeat the question, and they are able to explain the most important part of the question comprising of; what is asked? What is known? And what is the requirement?; 2) planning a problem solving, to answer a problem, students should plan what model is used to solve it, collect any existing information and connect it to some related facts and to some problems that have been learned; 3) solving the problem in accordance with the plan made and they should be sure that each step taken is correct; 4) reviewing the results to reinforce their knowledge and develop their ability in solving problems (Amir, 2015a; Yarmayani, 2016). When someone has discovered an answer to a problem he is working on either independently or with the help of others or using references from books or other sources, the question is not a problem anymore.

An essay test is a type of problem mostly used in researches because the form of this essay problem is easily written and is the best way to express one's ability in organizing thoughts and expressing knowledge completely. It is a type of test in which its learning advancement needs an answer with word descriptions (Irawati et al., 2014). An essay test is also able to train students to have reasons. It consists of two forms, namely: 1) an objective essay form is properly used for mathematics and science since its answer key is just one. The process of working this type of test is through certain procedures, and each step is given a score, and 2) a non-objective essay form, where the eval-

uation used tends to be affected by the subjectivity of the rater and this test form requires students' ability in conveying, arranging, choosing, and integrating ideas they have possessed through their own words (Suwarto, 2010).

An essay test carries its own strengths, namely: 1) it may use to evaluate student's comprehension at the highest level; 2) the students have some freedoms to choose, prepare and present ideas in their own words; 3) this kind of test may show students' strengths in organizing thoughts, supporting opinions, and creating ideas, methods and solutions; 4) the complexity of the questions in this kind of test and the thinking complexity expected from the student may be adjusted to the students' ages, ability and experiences; 5) teachers may understand their students in making answers and they do not only select the best answers from the alternative choices provided such as those in a multiple-choice test; 6) an essay test may also be used for a diagnostic test with a complex relation to the application of concepts, the analysis of problems or the evaluation of decisions; 7) this kind of test is easy to make. However, this kind of test also has some weaknesses, such as 1) different scores may be resulted in when it is evaluated by the same rater at a different time or different raters at the same time; 2) the length and complexity of the answer may cause a reliability problem in scoring; 3) it takes a long time to correct the answer sheet; 4) when the number of students is high in each class, it is almost impossible for the teacher to be familiar with each student's strengths and weaknesses, except if the teacher has a software for the purpose and can operate it; and 5) the cost needed to correct the essay test is very high (Suwarto, 2010).

In general, dealing with the correction of the essay test, Linn & Gronlund (1995) suggest five analogous pieces of advice namely 1) prepare the guideline of scoring; 2) read students' answers and compare them with the prepared guideline; 3) give suitable scores; 4) examine the whole students' answers in the same number, then continue to the next number; and 5) avoid any irrelevant factors in scoring. Linn & Gronlund (1995) give one more piece of advice namely: correct the answers without addressing the students' names and use two or more raters if the decision taken is important.

Therefore, it is vital for teachers to pose a form of problem in this essay test because a contextual problem enables students to relate it to their daily life. Moreover, this type of test may be used to evaluate various types of students' abilities, for example, their abilities in presenting ideas, thinking logically, and in making conclusions. For instance, in Relations and Functions materials, this test type is used to express a relationship among two or some objects such as family trees, states, and capital cities, students in a class based on their class rating, and the like.

## 2.2 ERRORS AND MISCONCEPTIONS IN MATHEMATICS

Learning is a continuous process involving students' active participation, although it clearly affects what they learn. Learning mathematics is cumulative, where new knowledge obtained deals with one's previous knowledge. (Roselizawati et al., 2014). According to NCTM, standard processes in basic abilities in mathematics include: 1) problem solving; 2) verification and reasoning; 3) communication; 4) connection, and 5) representation (Hasratuddin, 2014; Anggreini & Waluyo, 2017). The goal of learning mathematics in the content and mathematics learning in NCTM

is to develop one of the abilities in mathematics, namely the ability to make mathematical connections. Mathematical connections are the high-level thinking ability relating among concepts in mathematics either internally, namely the relations in the mathematics itself or externally, specifically the relationship between mathematics and other fields in daily life. The abilities in the mathematical connection among individuals are different, some with low, middle and high ones. By and large, there are three indicators of mathematical connection abilities: 1) writing mathematical concepts underlying the answers; 2) writing the relationship between objects and mathematical concepts, and 3) understanding problems in daily life in the form of mathematical models (Ni'Mah et al., 2017). Concerning such varied mathematical connection abilities, students do not always avoid their own errors when solving problems.

Error is a form of deviation from something considered to be either correct or systematic, consistent or incidental in a certain part (Agustia & Ikman, 2016; Andra & Ikman, 2016). Error is a form of deviation in something which is regarded as correct or a kind of variation in something which has previously been determined (White, 2010). Therefore, it can be concluded that an error in a variety of differences where one substantiates something which is believed to be correct as determined before. The cause of students' errors in working on mathematical problems is that students do not master a language; for instance, they do not understand statements in mathematical problems, do not know the meanings of words, do not master the concepts and the counting techniques well. The errors students often experience in solving problems are as follows: 1) understanding the problems; 2) determining what is asked in the problems; 3) solving mathematical models; 4) stating the end answers.

There are three types of errors. The first is a concept, where it is an abstract idea that enables students to be able to group an object in the form of instance and of not instance (Jusmiana, 2013). Error in concept in this present research is students' error in concept in this present research is students' or in distinguishing examples of functions and not functions, using formulas and not formulas for a problem. Indicators of an error of a concept are those in understanding the meaning of a problem, in translating a problem into a mathematical model, in the concept of the variables used to make a mathematical model, in the concept of making methods of eliminations and substitutions. The second is; principle it is an error students make since they do not write or have a mistake in writing theorems or formulas. The indicators of principle errors the researchers determined are those in: a) writing the formulas to determine the number of mapping; b) determining the principle of addition in the process of solving the equation of a variable; c) using the principle of multiplication in solving the equation of a variable; d) is not writing the answer; and e) writing an incomplete or unclear answer. The third is technique, where it is errors students make since they are not able to calculate properly. The indicator of this error the researchers determined is that the students cannot calculate correctly (Widodo, 2016; Wijaya & Masriyah, 2011).

Many factors that cause a student to make errors may be seen from the reasons why students have difficulties in learning. Students' learning difficulties in general may be discussed by cognitive and no cognitive factors. Cognitive factors cover students' intellectual ability in under-



standing mathematical materials in their minds, while non-cognitive factors are something outside their intellectual abilities such as their family backgrounds, economic conditions, health, and the like. In this present research, the factors of the error are from the students themselves, for example, the student's intellectual ability in understanding relations and functions problems.

Concept is an abstract idea enabling an individual to combine objects around their life into a form of example and non-example such triangle and no triangle, and the like. Concepts in a mathematical structure include pure, notational, and applied ones. A pure mathematical concept is a mathematical idea of grouping or combining numbers and relations among numbers for instance, seven, 9, XI, and I are concepts of odd numbers presented using different concepts. A concept in mathematical notation is the characteristics of numbers from the way they are presented. For example, 354 means that 3 is the hundreds number, added with 4 tens number, and with 9-unit number where it is caused by the position of the notation which determines the magnitude of numbers. While the concept in applied mathematics is the use of the pure concept and the concept in mathematical notation to solve mathematical problems.

The understanding of each individual is different and it is called conception. Although the concept has a clear meaning that has been agreed upon by mathematicians, students' conceptions are still different, and this happens due to the conceptions they have learned before. Conception may be formed from experiences in interpreting an event or another natural phenomenon. Conception is students' ability in interpreting an obtained concept (Suci & Purnomo, 2016). There are three criteria of students' conception, namely: a) understanding: any response which is appropriate with components determined although they are incomplete, any response the students give include intended component; b) misconception: any response the students give is illogical, the response shows understanding of a concept, but they also make errors in making a statement which is appropriate with the opinions of the experts; and c) not understanding: students do not give any response, repeat statements, and the response makes us irrelevant with the real answer (Haidar & Abraham, 1991).

A misconception is a concept where the concept is inappropriate with those recognized by the experts. A misconception is defined as a perception possessing meaning which is greatly different from that proposed by experts about certain topics or fields (Hammer, 1996). Moreover, a misconception is a kind of misunderstanding and misinterpretation coming from inaccurate meaning (Ojose, 2015). Therefore, from some definitions about conception, it can be stated that conception is an error in concept, different perception, or understanding in interpreting meaning from different opinions from those presented by previous experts.

Misconception refers to a concept that is inappropriate with that presented by the experts, or its form may be the initial concept of any error with incorrect relations among wrong concepts or ideas. The misconception may result from some causes, namely: the students themselves, the teacher, the textbook, the context, and the teaching methods. First is a misconception that is rooted in the students themselves arises because of their initial conception before they join in the lessons at school. If their early conception

contains misconceptions, the first conception will trigger misconceptions in further next materials. The second is the teacher. Some misconceptions may happen since the educator has a low mastery or incorrect understanding of the materials. The third is the textbook, where the most misconception comes from the textbook used in the learning process because 95% of learning activities in each level are centred on the textbook (Haidar & Abraham, 1991). A textbook that is too difficult may instigate misconception due to students' difficulties to understand its content; as a result, the student merely understand some concepts presented in the book; Fourth is the context. Students' daily life and experiences may trigger misconceptions. The language used in daily life, which is different from the language in the scientific field, will cause misconceptions. Fifth is the teaching method the teacher uses that may bring about students' misconception because of improper examples or applications of the concept learned, and also the use of properties that are inappropriate with the presented concept. The last is the knowledge formed from the students' ways of thinking which are influenced by their surrounding environment Masril (2012), Rosita et al., (2013).

There are some tools that may be used to detect any misconception. 1) Multiple Choice Test with Open Reasoning. This kind of test may be used to identify whether students have any misconception or just ignorance in solving a problem. 2) Diagnostic interview. It is intended to express students' ideas of concepts so that researchers know the reasons why students may have some misconceptions. Some examples of conceptions students experience in solving relations and functions problems are described as follows: Indicators of understanding a concept: restating a concept dealing with relations. There are four research subjects: FI, IN, NS, and SM. Each subject makes an error. In FI's error, it is stated that relation is combination, instead of a link. IN's error is that it is stated that a set is paired, and the pairing may be more than one. A member of a domain should be paired with a member of a co-domain instead of its set. NS is incorrect when he states that a set number may be paired with one. The correct statement is that a member of a set from a domain may be paired with more than one set member of a co-domain, and a set member may not merely be a number but maybe others. Any error made by SM is that he thought that relation is a set number that may be from two sets. The proper fact is that a set number is a member of the domain set and may be paired with more than one set member of the co-domain. The descriptions show one of the examples of misconceptions made by students about a concept related to relations. All subjects are incorrect in giving the meaning of relation. This shows that all subjects possess less understanding of the concept of relation (Irawati et al., 2014).

### 3. METHOD

The approach adopted in this present research was qualitative, and the type of the approach is descriptive in nature aimed at identifying any errors and misconceptions students conducted when solving Relations and Functions problems. The data were written answers. The data source was 23 students in grade VIII-A MTs Muhammadiyah Malang, Indonesia. The data examined were students' errors and misconceptions in Relations and Functions materials. In this research, the subjects were selected

using a purposive sampling technique where 6 students were taken as the research subjects on the basis of high, medium and low abilities groups where each group consisted of two students. The research was focused on the students' errors and misconceptions in solving Relations and Functions problems. Errors are obstacles students experience in solving problems.

The techniques of the data collection were as follows: 1) written administrated to the students, and 2) interviews with the students, intended to obtain information from the subjects dealing with their errors when they were doing the tests. Interviews were used to know the results of the learning process. The strength of interviews is that the researchers may directly meet the students so that the students may give the answers more freely and more deeply. The stages in the data analysis were as follows. First is data reduction, covering the following steps: a) correcting the students' test results to determine their scores categories such as high, medium and low, that would be used as the research subjects; b) the students' test results that would be intended to decide which students to be interviewed serving as the research subjects and c) the results of the interviews which were arranged into good and correct language arrangement. The second is data presentation, consisting of the arrangement of the student's test results and interviews. Three is conclusion drawing or verification (Kamariah, & Marlissa, 2016).

## 4. RESULT & DISCUSSION

### 4.1 ERRORS AND MISCONCEPTION ANALYSES OF HIGH SUBJECTS

On the basis of the data on the subjects' answer sheets and the results of their test results, two randomly taken subjects with high ability viewed from the table of the frequency and the percentage of the test results were determined, namely  $R_1$  and  $R_2$

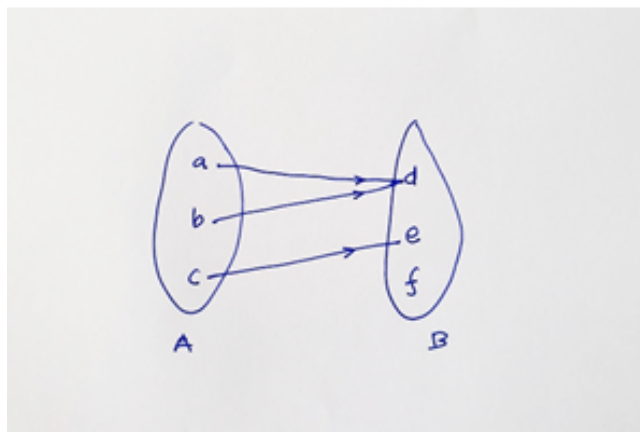


FIGURE 1.  $R_1$ 's answer

In Figure 1,  $R_1$  shows an example of a non-function instead of a function. In this case,  $R_1$  had some misconceptions in understanding the concept of function. On the contrary,  $R_2$ 's answer is presented in Picture 2.

At a glance, the answer given by  $R_2$  is not wrong, but actually, it is really incorrect since no labels are given in every circle in which each set member resides. Depicting a good function or relation in the form of an arrow diagram

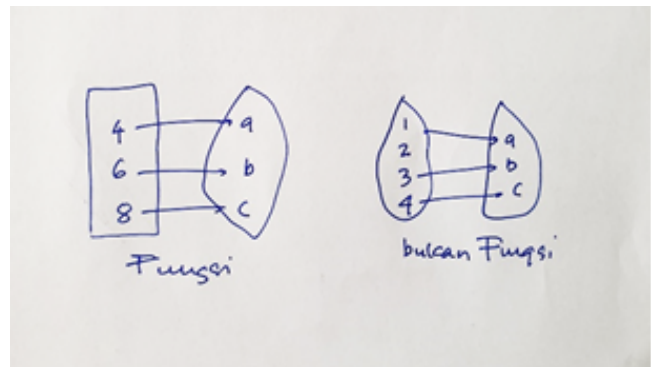


FIGURE 2.  $R_2$ 's answer

should 1) have two sets; 2) make circles as the divider between the domain and the co-domain sets; 3) give a label to each circle and 4) pair each set member of the domain with the sets of the co-domain.

The data above did not give enough evidence to show the place of errors and misconceptions, so that a more step is needed, namely triangulating the data of the test results using an interview method in order to reveal why subjects made errors or misconceptions when they answered the problem above. The following is the simplified results of interviews with each subject. The cause of the error made by  $R_2$  is that when presenting examples of relations or functions, actually he should give labels to facilitate some understandings in answering problem number two. But he forgot to write the labels so that he made mistakes. On the contrary, the reason why  $R_2$  had some misconceptions in doing problem number 2 is the order to make examples of functions and non-functions.  $R_2$  answered problem number 2 as depicted in his answer sheet since  $R_2$  assumed that a domain is not allowed to possess a pair in the co-domain, which is the same with other domain members. In fact, a function, of which the domain has a pair in the same co-domain as another domain member like the domain member a, has the co-domain, namely db, and this is also the case with the domain member b possessing the pair in the co-domain, namely d. It does not matter if a domain has the same member in the co-domain, which inevitably breaks the rule that a domain will possess more than one pair in the co-domain.

### 4.2 ERRORS AND MISCONCEPTION ANALYSES OF MEDIUM ABILITY SUBJECTS

Based on the data of the answer sheets and the test results from the randomly taken subjects, and also from the table of frequency and the percentage of their test results, two subjects with medium ability,  $R_3$  and  $R_4$ , were obtained

Figure 3 shows an error similar to that made by  $R_2$ , but  $R_3$  made an error when delineating circles in each set and did not give labels in each set. In problem number two, there are two instructions: state the relation in the form of an arrow diagram and show the relation in the form of a sequential pair. But,  $R_3$  merely wrote an answer in the form of an arrow diagram, and he did not include the answer of the sequential pair.

Respondent  $R_4$  made an error and had a misconception. The error is the same as that happened to problem number one. The misconception is that the respondent was incor-

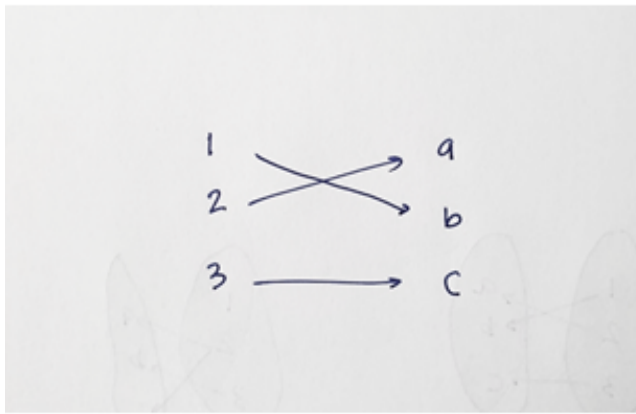


FIGURE 3.  $R_3$ 's answer

rect in giving an example of an incorrect function. In the figure, the answer A is an example of a not function. In this case,  $R_4$  had an error in the concept of a function, causing misconception. Figure 4 presents an error that is the same as that made by  $R_3$  and also  $R_1$  and  $R_2$ , where  $R_3$  did not give labels in each set. Moreover, A did not state the relationship in the form of a set of sequential pairs.

The data above could not be enough to show the positions of errors and misconceptions. Consequently, a more step is needed, namely making a triangulation of the data on the test results using interviews to reveal why the subjects made errors or had misconceptions when answering the problems above.

The results of the interviews are presented using an ethical approach, meaning that the results of the interviews are presented in the form of their essence as materials of the analysis, instead of using an emical approach, where the results of the interviews are transcribed, and some are cited. The essence of the results of the interviews with the subjects was taken by the researchers. Principally, the cause why  $R_3$  made an error is that he did not know if making a relational and functional figure should make circles for each set and give labels for each set. What  $R_3$  just knew is just pairing each member of a domain set and the member of the co-domain set. As a result,  $R_3$  had misconception when he presented an example of a function since he often misunderstood that a function is that each member of a co-domain had a pair, instead of that a function is that each member of its domain should have its own pair with

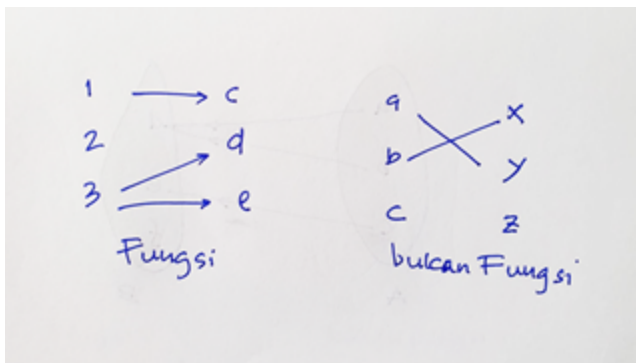


FIGURE 4.  $R_4$ 's answer

the member of its co-domain. For that reason, A had misconceptions in interpreting a concept of a function.

The reason why  $R_4$  made an error in depicting a relation and a function like those made by other subjects is that he forgot to give labels to each set and did not know that when drawing a relation and a function, different curves should be used. But he combined them in one curve. Concerning the misconception held by  $R_4$  when he calculated for example,  $i(1)=a(1)+1=a+1=2$  dan  $f(3)=a(3)-5=-2$ , he still often thought that the  $a1$  and the  $1$  is the same. In fact, the  $a1$  and the  $1$  are two different terms that cannot be directly operated. The rule is that if one wants to add the  $a1$  and the  $1$ , one firstly should find the value of the  $1$ . When making some calculations in the additional form as presented in his answer sheet, namely  $3*2=5$  dan  $3*3=11$ ,  $R_4$  did not focus his attention on the calculation and did not review it. As a result,  $R_4$  had misconceptions about the multiplication and addition of different terms.

### 4.3 ERRORS AND MISCONCEPTION ANALYSES OF LOW ABILITY SUBJECTS

#### 4.3.1 SUBJECTS WITH LOW ABILITY

Based on the data on the answer sheets from the subjects and the results of the test, two subjects were determined, namely  $R_5$  and  $R_6$ .

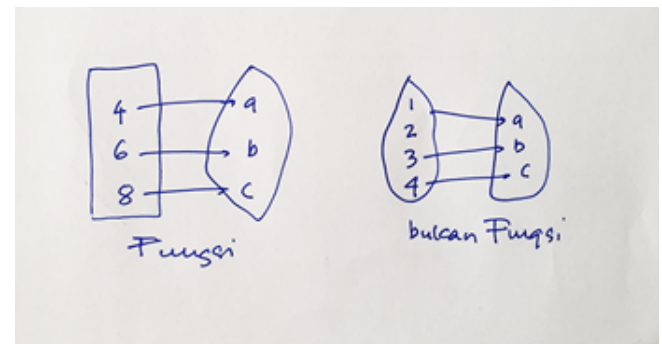


FIGURE 5.  $R_5$ 's answer

Figure 5 shows the  $R_5$ 's answer where the error made by:  $R_5$  is the same as that by other subjects, namely they forgot to give labels to each set and in the problem number 1,  $R_3$ ,  $R_4$  and also  $R_5$  also made errors that they did not state it in the form of a sequential pair set.

Some efforts were made to reinforce the research results showing the position of errors and misconceptions. Some interviews were conducted to help analyze why the subjects made errors or had misconceptions when they answered the mathematical problems above.

As stated above, the interviews were described using an ethical approach, and the results showed that the cause why  $R_5$  and  $R_6$  made errors when they stated a set in the form of an arrow diagram is that because they did not know if each set should also be given labels and the subjects, in stating the sequential pair set, had difficulties in writing the sequential pair set so that they did not show it in their answer sheets. In terms of misconception,  $R_5$  employed the wrong formula.  $R_5$  stated that he did not yet understand how to find the number of mappings for a set. Hence, the subject thought that the formula he employed had been correct already. Another misconception is that  $R_5$  was not



careful in the calculation so that he made an error where the value of  $a = -1$ , instead of  $b = 1$ .

The cause why  $R_5$  made an error when he stated a relation or a function in the arrow diagram is that he did not know that when expressing a relation and a function, labels for each set should be provided. The subject thought that in a relation and a function, it is not necessary to give labels in each set; what is important is that each domain set should have pairs in the co-domain set. The reason presented by the subject why he did not state it in the form of a sequential pair set was that he was not careful in reading the instruction in the problem. The next error is that the subject did not recognize what to do next after he knew the number of mappings from the domain to the co-domain. Therefore,  $R_5$  merely wrote the formula, and this error also occurred to other subjects.

On the basis of the descriptions above, high numbers of errors and misconceptions caused by the subjects are that they had low understanding in stating a relation or a function well. Moreover, the students also forgot to give labels when stating a relation or a function in the arrow diagrams. They thought that if they did not write labels in each set, he did not make errors. Moreover, they were also less careful in reading the instruction in the problems. Some subjects also showed the same errors, namely when stating examples of functions or not functions, they still could not distinguish which one is function and is not function. They also had problems in the process of calculation, such as errors, in addition, reduction, multiplication and also division.

These research results confirm the result presented by Irawati et al. (2014) that misconceptions exist in some errors in presenting a concept and giving examples of a concept. This present research also showed that students have difficulties in understanding a concept, principle (the use of formula) and numerical (calculation results), and verbal aspects (Sari & Zuzano, 2011). These research results are also in line and reinforce the research results stating that students with high ability do not make errors in solving problems, those with the medium ability make errors in the concept namely in determining the relations and certain values fulfilling a function, while those with low ability show all types of errors; they do have not been able to state relations correctly in the form of sequential pairs, to determine the domain and the co-domain, and to use proper procedures in solving problems. Kamariah, & Marlissa (2016); White (2010).

These results also showed that errors that happen in solving problems are caused by the fact that subjects did not review their answers. This supports In'am's research result (2014) that in problem-solving using the Polya approach; the fourth aspect deals with the review of the answer, which was rarely conducted by students due to things namely those with high ability feel sure that their answers have been correct, while those with low medium and low abilities do not have enough time to review their answers.

## 5. CONCLUSION

Based on the research results and discussion in identifying errors and misconceptions in solving problems on Relations and Functions materials for the Grade VIII-A MTs Muhammadiyah 1 Malang, some conclusions are drawn: 1) subjects with high ability merely make one error and one misconception, where the error is an improper way in stat-

ing a relation or function in the proper form of an arrow diagram and misconception in showing the example of functions and not functions; 2) subjects with low ability made errors where they less properly state relation and function in the form of an arrow diagram, where they do not state it in the form of a sequential pair, errors in the calculation, and in using formulas. These technical and principal errors may be called misconceptions; 3) subjects with the low ability make all types of errors. They have not been able to state relations or functions properly, say them in the form of sequential pairs, use formulas improperly, make errors in making calculations that determine the results. The factors causing students to make errors and misconceptions are as follows: students' low mastery of the prerequisite materials and less proper ways of learning, their low habits in training their cognitive ability by working on exercises dealing with relations and functions, and also the ways the teachers do in conveying relations and functions materials.

Based on the results of the research as mentioned above, it can be written recommendations for teachers so that in learning relations and functions they can provide a better understanding through contextual-based learning so that students' understanding of relations and functions does not occur misconceptions about the material being studied. It should also be stated that this research is limited to the material taught in grade 8 so that the results can only be generalized to subjects that have relatively the same characteristics.

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